

Graphing Rational Functions Notes

Function:

The function is a relation such that no two ordered pairs have the same first element. A function may be denoted as $y = f(x)$ which is read "f of x". A function may be written as $f: x \rightarrow y$, where $x \in \text{domain}$ while $y \in \text{range}$.

Sample Problem 1: Find the Range of the following rational function.

1. If $f(b) = \frac{b-b^2}{1+b^2}$, find a. $f\left(-\frac{1}{2}\right)$; b. $f(-2)$; c. $f(-1)$.

$$a. \frac{\left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^2}{1 + \left(-\frac{1}{2}\right)^2} = \frac{-\frac{3}{4}}{\frac{5}{4}} = -\frac{3}{4} \times \frac{4}{5} = -\frac{12}{20} = -\frac{3}{5}$$

$$b. \frac{(-2) - (-2)^2}{1 + (-2)^2} = \frac{-2 - 4}{1 + 4} = \frac{-6}{5} = -1\frac{1}{5}$$

$$c. \frac{(-1) - (-1)^2}{1 + (-1)^2} = \frac{-1 - 1}{1 + 1} = \frac{-2}{2} = -1$$

Sample Problem 2: Draw the graph of the following rational function.

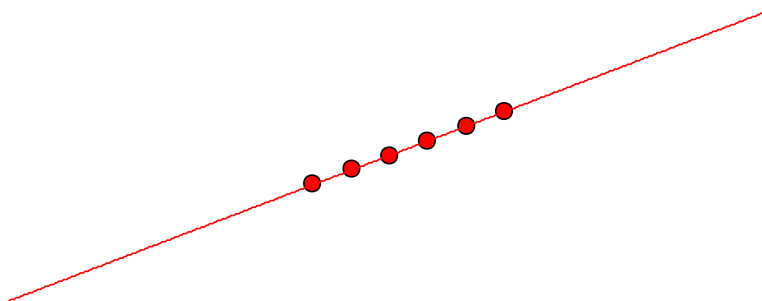
2. Below is the table containing the domain of $f(x) = \frac{x}{2}$, find $f(x)$ given the value of x below and graph its function.

X	-2	-1	0	1	2	3
Y	-1	-1/2	0	1/2	1	3/2

$$f(-2) = \frac{-2}{2} = -1 \quad f(-1) = \frac{-1}{2} \quad f(0) = \frac{0}{2} = 0 \quad f(1) = \frac{1}{2} \quad f(2) = \frac{2}{2} = 1 \quad f(3) = \frac{3}{2}$$

Plot the points then draw the graph.

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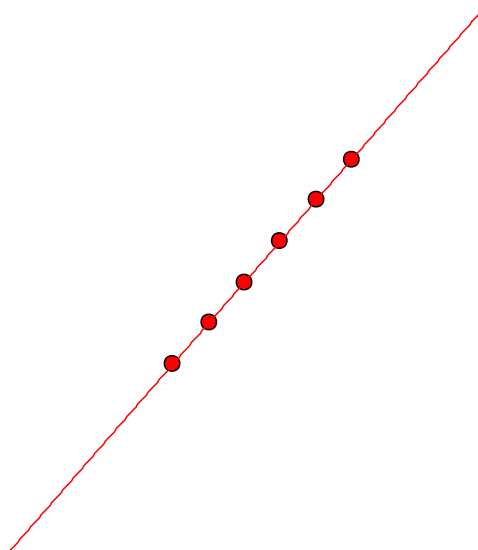


3. Below is the table containing the domain of $f(x) = \frac{3x}{2}$, find $f(x)$ given the value of x below and graph its function.

X	-2	-1	0	1	2	3
Y	-3	-3/2	0	3/2	3	9/2

$$f(-2) = \frac{3(-2)}{2} = -3 \quad f(-1) = \frac{3(-1)}{2} = -\frac{3}{2} \quad f(0) = \frac{3(0)}{2} = 0 \quad f(1) = \frac{3(1)}{2} = \frac{3}{2} \quad f(2) = \frac{3(2)}{2} = 3 \quad f(3) = \frac{3(3)}{2} = \frac{9}{2}$$

Plot the points then draw the graph.



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Graph of Rational Function

The line $x = a$ is a vertical asymptote if the graph increases or decreases without bound on one or both side of the line as x closer to closer to $x = a$.

The line $y = b$ is a horizontal asymptote if the graph approaches $y = b$ as x increases or decreases without bound. Note that it doesn't have to approach $y = b$ as both increases and decreases. it only need to approach it on one side in order for it to be a horizontal asymptote.

1. The graph will have a vertical asymptote at $x = a$ if the denominator is zero at $x = a$ and the numerator isn't zero at $x = a$.
2. If $n < m$ then x axis is the horizontal asymptote.
3. If $n = m$ then the line $y = a/b$ is the horizontal asymptote.
4. If $n > m$ there will be no horizontal asymptote.

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

Sample Problem 3: Draw the graph of the following rational function.

4. Sketch the graph of a function $f(x) = \frac{3x+1}{2x-1}$

Y-Intercept

$$f(0) = \frac{3(0)+1}{2(0)-1} = -1$$

X- Intercept

$$3x+1=0 \quad x = -\frac{1}{3}$$

Asymptote

$$2x-1=0 \quad x = \frac{1}{2}$$

Horizontal asymptote

$$a = 3, b = 2 \quad y = \frac{a}{b} = \frac{3}{2}$$

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$$f(x) = \frac{3x+1}{2x-1}$$

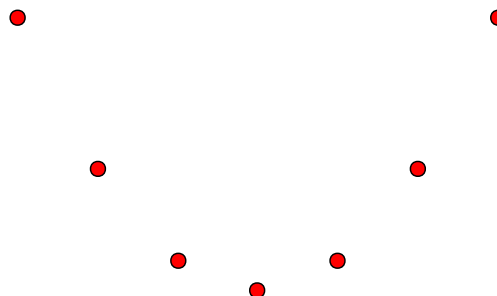
Sample Problem 4: Identify the function and the range of the following graph.

5. Find the domain and range of the graph below.

$$f(x) = \frac{x(x)}{2}$$

$$f(x) = \frac{x(x)}{2}$$

Series 1



Ordered pair : $\{(-3, 9/2), (-2, 2), (-1, 1/2), (0, 0), (1, 1/2), (2, 2), (3, 9/2)\}$

Domain: $\{.....-3, -2, -1, 0, 1, 2, 3 ...\}$ all positive and negative real numbers

Range: $\{0, 1/2, 2, 9/2 ...\}$ all positive real numbers